Split-Plot General Multilevel-Categoric Factorial Tutorial

Introduction

In some experiment designs you must restrict the randomization. Otherwise it wouldn't be practical to perform the runs. For example, temperature cannot be easily changed in many applications. The solution may be a "split-plot" design, which originated in the field of agriculture. Experimenters divided large areas of land, called "whole plots," into smaller "subplots" that could be treated separately. For example, they planted several crop varieties—one each per whole plot, and then applied different fertilizers on each subplot.

The analysis of a split-plot design is tricky, even for statisticians. But Design-Expert® version 9 software makes it easy.

To illustrate how Design-Expert does design and analysis of split plot, let's follow an example from Montgomery's *Design and Analysis of Experiments*. Feel free to skip the boxed bits—these are sidebars for those who want to know all the details and spend more time to explore things.

> ⌧ *Explore details on this case study*: Refer to Section 14.4 "The Split-Plot Design" in the 8th Edition of the book referenced above—published by Wiley in 2012—for more details on the experiment's structure and this particular case. It's also covered in Stat-Ease's computer intensive workshop on **Split-Plot Design for Hard-to-Change Factors**.

Background for Case Study

A manufacturer wants to determine the effect of three methods of pulp preparation and four levels of cooking temperature on the tensile strength of paper. These twelve combinations can be conducted in one day.

To increase statistical power, the paper chemist decides to perform three replicates of this twelve-run general factorial design over three days, which produces a total of 36 runs. However, it is not feasible to perform the experiment in a random fashion. Instead, on each of the three days, this experimenter produces a batch of pulp and divides it into four samples to be cooked at four different temperatures. The process is repeated for pulp prepared by the two other methods.

A flowchart of the experiment is shown below.
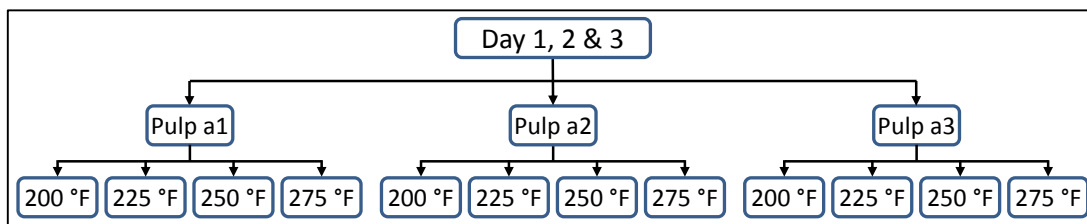
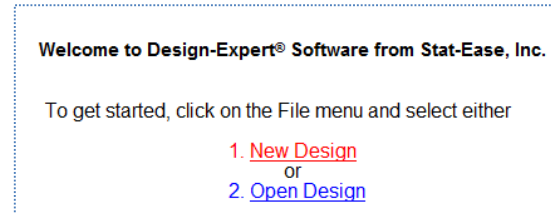*Flowchart of split plot experiment on papermaking process*

In statistical terms, the split plot experiment can be structured as:

- Whole plots for the three batches of pulp (hard-to-change factor)

- Sub-plots for the four samples cooked at four different temperatures (easy to change factor).

You will set up this design as a blocked (by day) split-plot general factorial.

Design the Experiment

Let's build the design. This is done via the usual procedure for general factorials, with days accounted for as blocks. From the Design-Expert welcome screen, click **New Design** (or click the blank-sheet icon 🗋 on your toolbar, or choose File, New Design).

Welcome to Design-Expert® Software from Stat-Ease, Inc.

To get started, click on the File menu and select either

1. New Design
or
2. Open Design

Opening a new design

Then from the default **Factorial** tab under **Split-Plot** click **Multilevel Categoric**. Choose **2** as the number of factors. Click the **Vertical** radio button. Then enter **Pulp Prep** as your Factor A name. It's already set up a Hard-to-Change factor so leave that be. Mouse down to the **Levels** column and type in **3**. Then enter **a1**, **a2**, and **a3** as your **L(1)**, **L(2)**, and **L(3)** treatment names.

Still on the same screen, name your second factor **Temp**, with **deg F** for units, **4** for the number of levels, and **200**, **225**, **250**, and **275** as your treatments. This factor is Easy to Change as already set by the program. For **Type** select **Ordinal**, thus denoting its numeric nature.

Your screen should now match that shown below.
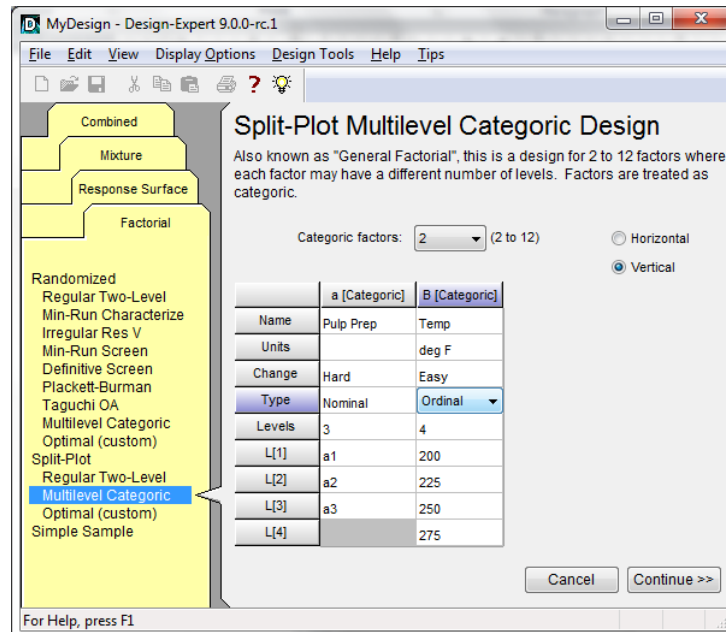
Last factor changed to "ordinal"

Press **Continue** and for **Replicate** enter **3** and check on (✓) **Assign one block per replicate** to produce 36 runs (36=3x3x4 levels from the three factors) in 9 groups.

Three replicates with one block per replicate

Again press **Continue**. Now for **Block Names** enter **Day 1**, **Day 2** and **Day 3**.

Block names

Continue and for the one response **Name** enter **Tensile** in **Units** of **strength**. Then to assess the power of this design, type in **Delta** of **6** and **Sigma** of **2**.

Entering response name and units, plus the signal to noise ratio

Next click **Options** and change the **Variance ratio** to **0.25** and the model **Order** to **2FI** as shown below.

⊠ *Explore background on variance ratio*: Design-Expert defaults to a variance ratio of 1 for group (whole plot) to residual variance. But in this case, the whole-plot factor, although hard to change, can be controlled with little variation relative to the sub-plot factor of temperature.

P.S. The changing of the order presumes that the experimenters must be interested in the interaction of the pulp prep with temperature, otherwise why bother doing it at all. Therefore they want to know if this model term (aB) can be estimated with a 'decent' (80% being generally acceptable) level of power.

Changing options for power calculation

Press **Continue** and review the power for the design. Notice that, due to the restriction in randomization in the split plot, the power for seeing effects of the hard-to-change factor "a" (Pulp prep) is reduced, whereas the power for see the effects of the easy-to-change factor "B" (Temperature) and its interaction "aB" are increased, albeit only slightly.

⚙ *Explore a tip about grouping effects from split plots*: Due to the structure of this design, interactions of hard to change (whole plot) factors with ones that are easy to change (sub-plot) must be tested against the sub-plot error.

Power is reported at a 5.0% alpha level to detect the specified signal/noise ratio.

Recommended power is at least 80%.

The ratio of whole-plot to sub-plot variance is 0.25 (default is 1).

Power is shown for the current split-plot design.

For comparison, the power of an equivalent completely randomized design is also given.

Tensile   strength

Signal (delta) = 6.00   Noise (sigma) = 2.00   Signal/Noise (delta/sigma) = 3.00

	a	B	aB
Split-Plot:	86.5 %	99.9 %	91.9 %
Randomized:	99.9 %	99.6 %	87.8 %

Results for power

Notice the loss in power for the hard-to-change factor "a" (Pulp Prep) still leaves it above the recommended minimum. ☺ **Continue** to complete the design-building wizard. Design-Expert now displays your design in groups by level of the hard-to-change factor "a" (Pulp prep), within which the easy-to-change factor "B" (Temperature) is laid out in completely random order.


Design built for split plot (run order randomized so your screen probably differs from this)

At this stage the design would normally be printed as a run-sheet for the actual experiment, with resulting responses of tensile strength recorded.

Analyze the Results

To avoid the trouble of typing in data, simply <u>read in the responses</u> via the 📂 icon or via File, Open Design from the main menu.
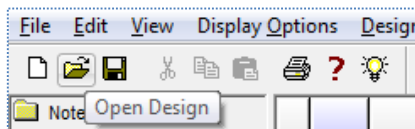
*Opening the design file to get the response data*

Select the file named **Pulp.dxpx**. If this file does not appear, download the tutorial files by going to the Help Menu—Tutorials.

The analysis of a split plot design must be done differently that a completely randomized experiment. To get the correct statistical tests for each, Design-Expert creates two separate ANOVA's for:

- whole plot treatment (pulp prep), and

- subplot treatment (temperature) and the interaction between whole plot and subplot individually.

Then it fits the full model in order to get meaningful diagnostics and model graphs.

Start by clicking the Analysis node labeled **R1:Tensile** which can be found in the tree structure along the left of the main window. Then press ahead to the **Model** tab displayed in the progressive toolbar. Design-Expert preselects terms for the designed-for two-factor interaction (2FI) model—these are labeled "M". Enable the program to check if any terms can be safely taken out by changing the **Selection** to **Backward** as shown in the screen shot below.
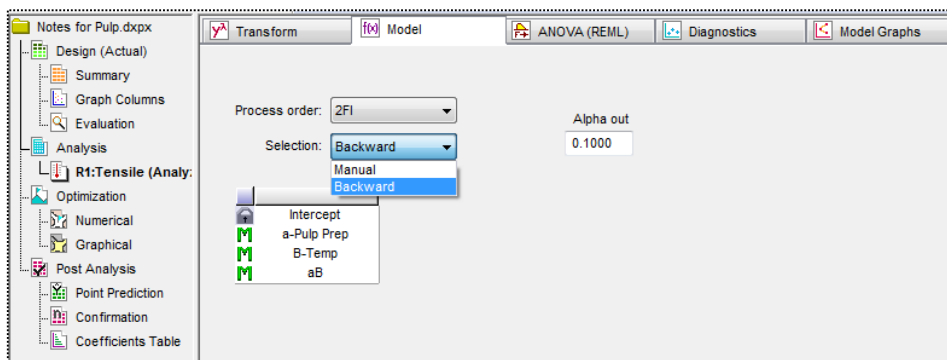

Model selection

Any terms that exceed the "Alpha out" criterion (p-value of 0.1 by default) will be removed and re-designated to Error ("e") f after looking over the statistics, you want to remove a term, simply go back and double-click it (or right-click for a menu of designation choices).

⊠ *Heads-up on effects selection for general-factorial split plots*: Due to the structure of these designs, the handy graphical half-normal plot of effects is not produced like it would be for an experiment with factors only at two levels. Instead, run the backward selection algorithm—details for which can by clicking Screen Tips (the light-bulb).

Continue to the **ANOVA (REML)** tab and review the p-values for significance. In this case all the designed-for model terms survive the elimination process, that is, they are all statistically significant.

> ⌧ *A note about terminology*: As you probably know, ANOVA is the acronym for analysis of variance—the fundamental approach to assessing model significance for all DOE where runs can be completely randomized. However, in a split plot the components of variance are estimated by a different approach called REML, which stands for restricted maximum likelihood estimation. Sometimes, depending on the structure of the design, REML agrees with ordinary-least-squares ANOVA exactly, but often the results differ somewhat.
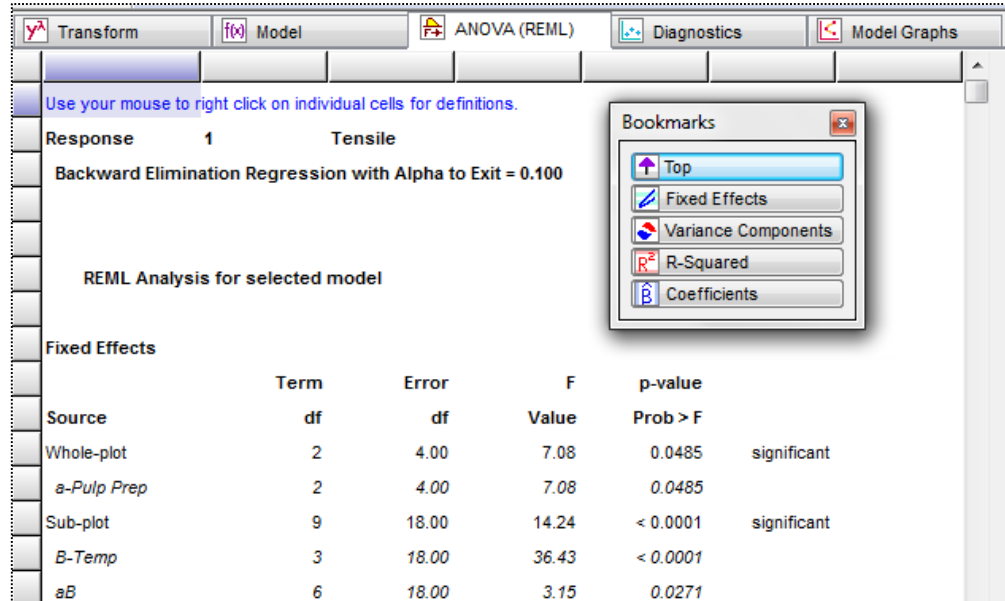
Results of ANOVA (REML)

> ⌧ *Details on statistics presented in ANOVA (REML) report*: Search Help, Contents for much more information, including references, on everything you see presented on this screen. Since, in the interest of keeping things to the point, these tutorials only provide the highlights, we must move on.

Move on to the next phase of analysis—verifying the statistical assumptions—by pressing the **Diagnostics** tab. First up is the normal plot of the residuals—internally studentized to provide the most effective view of any obvious discrepancies. There are no obvious irregularities in this case—the points fall in line as expected from normal data.
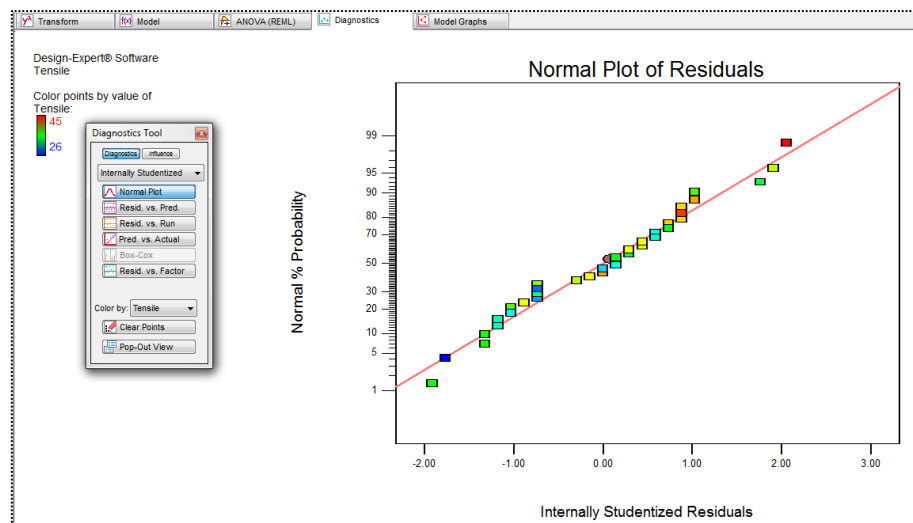

Diagnosing the normal plot of residuals

Examine all the graphs available via the Diagnostics tool. You should see nothing abnormal.

> ⌧ *Heads-up on diagnostics for split plots*: Due to the structure of these designs, the handy Box-Cox plot for response transformations us not produced like it would be for a fully-randomized experiment. Also, as you may have noticed, the option for externally studentizing residuals does not appear. Nor do other deletion diagnostic tools such as Cook's Distance, DFFits and DFBetas. These are prices to pay for accommodating hard-to-change factors in an experiment design.

Press ahead to **Model Graphs**. On the **Factors Tool** <u>right click</u> over the **C: Temperature** bar and change it to the **X1 axis**.
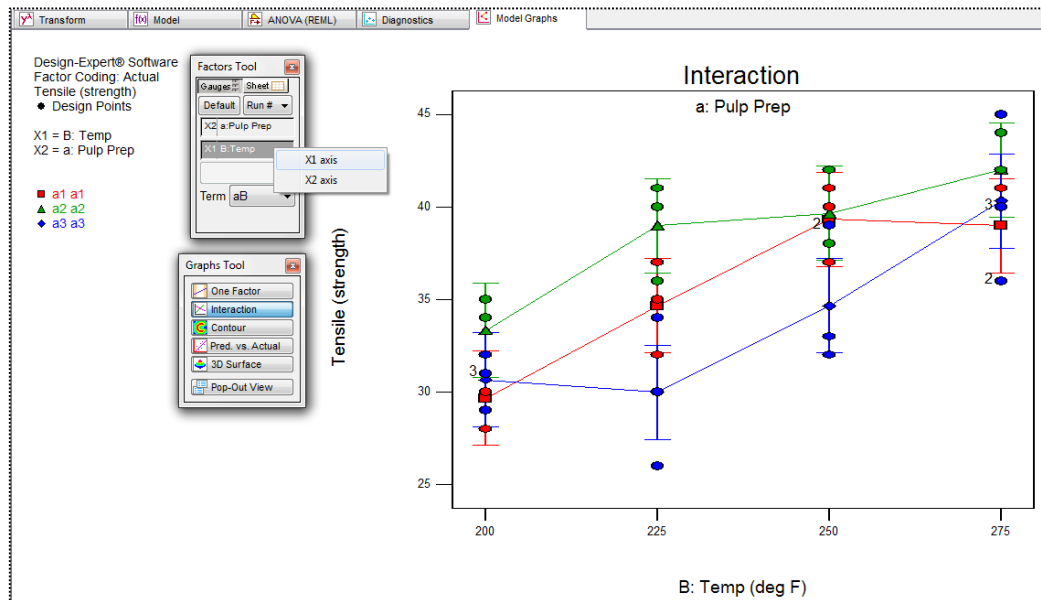
Changing axes

You now see an interaction plot of the continuous factor temperature at three discrete levels of pulp preparation. From this graph we can see pulp preparation B2 (symbolized by the green triangles) will maximize tensile strength. Evidently it will be most robust to temperature changes, especially in the temperature range of 225 to 275 degrees F.

Conclusion

This is an ingenious application of designed experimentation using a split plot structure to handle the hard-to-change pulp preparation. Design-Expert provides valuable insights on how power is reduced by restricting randomization. In this case the actual effect of pulp prep (factor "a") far exceeded that of minimal importance so the experiment succeeded.